Internal resonances in nonlinear nanocantilever arrays under electrostatic actuation

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INTERNAL RESONANCES IN NONLINEAR NANOCANTILEVER ARRAYS UNDER ELECTROSTATIC ACTUATION

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ABSTRACT

The nonlinear dynamics of nanoelectromechanical cantilever arrays is investigated using a comprehensive analytical multiphysics model that takes into account geometric and electrostatic nonlinearities. In particular, the internal resonances between the different cantilevers are analyzed using a multi-modal Galerkin discretization coupled with a perturbation technique. Such systems offer a perfect mechanical synchronization, interesting nonlinear behaviors and exchange of energy between their different components which makes them potential candidates for multi-mass sensing applications.

1 INTRODUCTION

Despite the fact that internal resonances (IR) have interesting dynamic properties, many designers strive to avoid it in their models. In recent years, this phenomenon is more and more used due to its property of suppressing oscillations in cantilever [1] or also enhancing the coupling effect in dynamical systems [2]. Sethna [3] was one of the first researchers who investigated the phenomenon of IR. He studied the influence of quadratic nonlineriarities in a system which exhibits two-to-one (2:1) internal resonances with subharmonic parametric resonance. He showed that the steady-state motions are more interesting when the system possesses external and internal resonances. Moreover, several studies were performed regarding resonant response of system under harmonic excitation forces [4, 5]. Nayfeh and Mook [6] have given a complete treatment of this subject. A common result from these researches is that the IR ratio depends on the type of coupling of the system. For quadratic coupling two-to-one (2:1) IR ratio can be used and for cubic nonlinearities both one-to-one (1:1) and three-to-one (3:1) IR ratios give amplitude modulated response. Recently, Gutchemidt and Gottilieb [7] modeled a continuum initial-boundary-value problem of a doubly-clamped microbeam array excited at several DC biases and periodic AC voltages. They showed that, for DC near systems first pull-in instability, three-to-one internal and combinational resonances were identified.

Motivated by the previously cited researches, and aware about the important influence of the internal resonances (IR) in nonlinear systems such as NEMS, this work is conducted to investigate the IR that can occur in a device composed by one or several coupled nanocantilevers under an electrostatic actuation. In this context, two types of configurations are considered: the first one, used by Kacem *et al* [8], is a single nanocantilever actuated by an electrosatic force while the second configuration consists of an array containing N coupled nanocantilevers actuated by a single electrode. The two systems are represented in Figure 1.

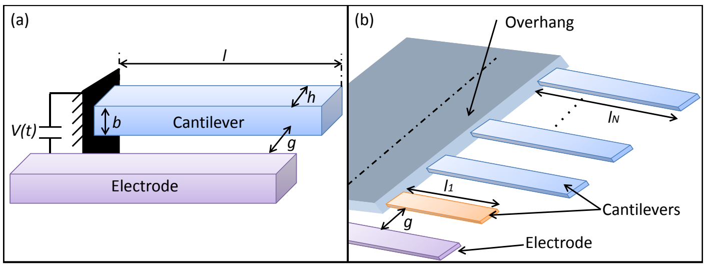

Figure 1: (a) A single nanocantilever electrostatically actuated. (b) An array of N nanocantilever beams. Only the first cantilever is excited by the electrode and $l_N = r * l_1$ with $r > 1$.

2 MODEL

As a first step, a single nanocantilever (Figure 1 (a)) is considered. In order to identify the internal resonance conditions, a multi-modal Galerkin method is used to calculate the natural frequencies of the three lowest bending modes. Figure 2 (a) shows the variation of these frequencies with respect to the DC voltage. After a meticulous investigation of the possible IR, and despite the fact that there are many possibilities of internal resonances and exchange of energy among higher and lower order modes, two major issues are noticed: the study of such systems gives a very complex set of amplitude and phase equations and above all, it is arduous to obtain in experiments such relations between the different modes.

In order to avoid these complexities, another approach is considered. It is inspired from [9] and consists of an array of N nanocatilevers depicted in Figure 1 (b). The dynamics of the considered system is modeled using a set of coupled nonlinear partial differential equations.

A reduced-order model is generated by modal decomposition transforming the equations of motion into a finite degree- of-freedom system. Then, the method of multiple scales is used as a direct attack of the resulting equations while assuming the dominance of the first bending mode for each cantilever. This permits deriving the phase and amplitude modulation equations.

For instance, we consider the case of two coupled cantilevers. The variation of the natural frequency of the first bending mode of each beam with respect to the DC voltage is presented in Figure 2 (b). The internal resonances 2:1 and 3:1 between the two cantilevers are possible for particular values of V_{dc} given by the intersection points of the corresponding curves. Figures 3 and 4 display simulated frequency-response curves of the considered device for 3:1 IR and two set of design parameters. Remarkably, the model enables the capture of the transition from softening to hardening behavior for the second cantilever.

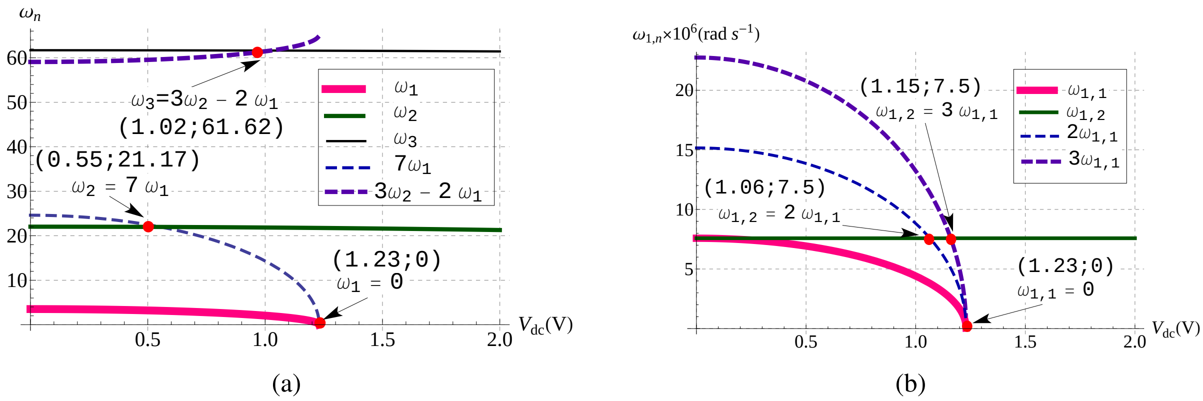

Figure 2: (a) Variation of the dimensionless natural frequencies of the first three bending modes for a single nanocantilever with respect to the driving voltage V_{dc}. 7:1 and a combinational $(\omega_3 = 3\omega - 2\omega_1)$ IR are identified. (b) Variation of the natural frequencies of a two-beam system with respect to the driving voltage V_{dc}. 2:1 and 3:1 IR are identified.

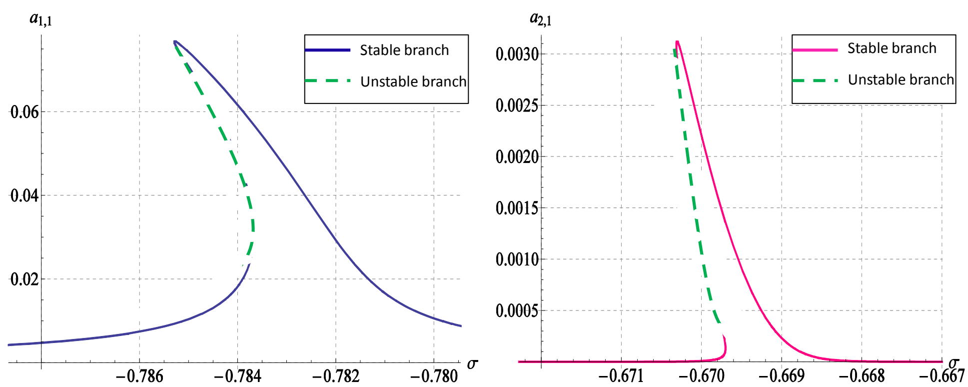

Figure 3: Variation of the modal amplitude $a_{1,i}$ of each cantilever with respect to the detuning parameter σ in the case of 3:1 IR, for $V_{ac} = 1\,mV$, $r = 2$, $l_1 = 9\,\mu$m, $h = 100\,nm$ and $b = g = 200\,nm$. The quality factor is $Q = 5000$ and the coupling parameter is $d = 3$.

3 CONCLUSION AND PERSPECTIVES

A simple electrostatic actuation of a nano-beam can provoke progressively the oscillation of several coupled nonlinear cantilevers due to internal resonances. This effect can be beneficial for multi-mass detection by providing a perfect control of the synchronization via the mechanical coupling between the cantilevers.

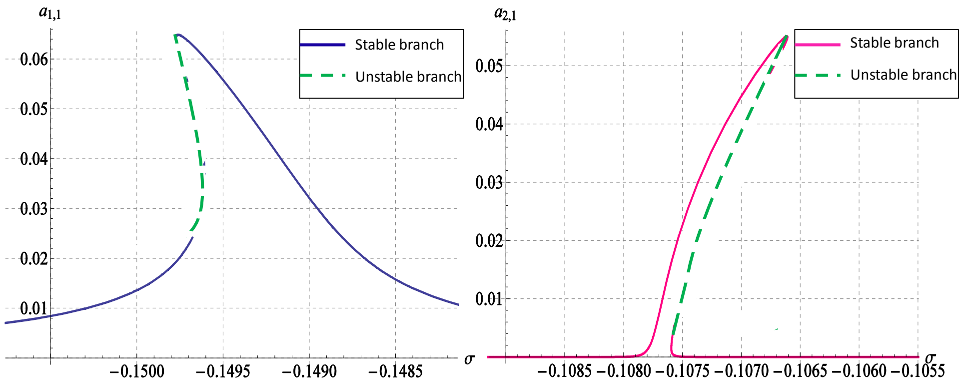

Figure 4: Variation of the modal amplitude $a_{1,i}$ of each cantilever with respect to the detuning parameter σ in the case of 3:1 IR, for $V_{ac} = 10\,mV$, $r = 4.5$, $l_1 = 3\,\mu$m, $b = 200\,nm$, $h = 110\,nm$ and $g = 120\,nm$. The quality factor is $Q = 10000$ and the coupling parameter is $d = 0.5$.

REFERENCES

[1] K. L. Tuer, M. F. Golnaraghi, and D. Wang. Development of a generalised active vibration suppression strategy for a cantilever beam using internal resonance. *Nonlinear Dynamics*, 5(1):131–151, 1994.

[2] A. Q. Siddiqui and M. F. Golnaraghi. Vibration suppression in flexible gyroscopic system using modal coupling strategies. *Mathematical Problems in Engineering*, 2(2):107–129, 1996.

[3] P. R. Sethna. Vibrations of dynamical systems with quadratic nonlinearities. *Journal of Applied Mechanics*, 32(1):576–582, 1965.

[4] B. Balachandran and A. H. Nayfeh. Observations of modal interactions in resonantly forced beam-mass structures. *Nonlinear Dynamics*, 2(2):77–117, 1991.

[5] V. Kumar, J. K. Miller, and J. F. Rhoads. Nonlinear parametric amplification and attenuation in a base-excited cantilever beam. *Journal of Sound and Vibration*, 330(22):54015409, 2011.

[6] A. H. Nayfeh and D. T. Mook. *Nonlinear Oscillations*. Wiley, New York, 1979.

[7] S. Gutschmidt and O. Gottlieb. Bifurcations and loss of orbital stability in nonlinear viscoelastic beam arrays subject to parametric actuation. *Journal of Sound and Vibration*, 329(1):3835–3855, 2010.

[8] N. Kacem, J. Arcamone, F. Perez-Murano, and S. Hentz. Dynamic range enhancement of nonlinear nanomechanical resonant cantilevers for highly sensitive nems gas/mass sensor applications. *Journal of Micromechanics and Microengineering*, 20(4):9, 2010.

[9] S. Gutschmidt and O. Gottlieb. Nonlinear internal resonances of a microbeam array near the pull-in point. EUROMECH Conference ENOC, 2008.